Comment on "A Theory of Three-Dimensional Parachute Dynamic Stability"

W. R. Graham* and B. R. Moss†

University of Cambridge, Cambridge, CB2 1PZ, UK

The subject of this comment is a pioneering early study of parachute stability.[1] Although published almost fifty years ago, it continues to be cited by current researchers in the field. Its non-linear, time-domain, simulations may have been superseded by more recent computations, but the insight provided by its linearized formulation remains valuable.

Unfortunately, the paper as published contains some inconsistencies in the formulae describing the 'longitudinal' (symmetric) motions of the gliding parachute. This comment aims to establish whether these are simply typographical errors, or mistakes that affect the reliability of the results. It is intended to be read in conjunction with the original publication.

The first correction is to the term in θ' in Eq. (14). This currently reads

$$\left[\sin\alpha_0 - \frac{C_{T\alpha}\cos\alpha_0 + 2C_{T0}\sin\alpha_0}{2k(1+r_c)}D\right]\theta'; \tag{1}$$

its corrected form is:

$$-\left[\sin\alpha_0 + \frac{C_{T\alpha}\cos\alpha_0 + 2C_{T0}\sin\alpha_0}{2k(1+r_c)}\right]D\theta'. \tag{2}$$

With this emendation, Eqs. (20) for the coefficients of the characteristic equation follow, *except* for the formula for E_3. This should read:

$$E_3 = \frac{C_{T0}\left[k + i(1+r_c)^2\right] + k^2(1+r_c)}{2k^2 i(1+r_c)^2}. \tag{3}$$

Finally, there is a sign error in the expression for the derived coefficient K_2, introduced

*Senior Lecturer, Department of Engineering, Member AIAA.
†Undergraduate Student, Department of Engineering.

in Eq. (23). This quantity is in fact given by

$$K_2 = E_5(E_2E_3 + E_1E_4 - E_5) + E_1(E_3E_6 - 2E_2E_7). \tag{4}$$

In addition to these corrections, three clarifications should be noted:

- the hydrodynamic moment-of-inertia factor B_i introduced in Eq. (26) is referred to the canopy centre of gravity (CG), whereas its dimensional counterpart I_{ch} is implicitly defined, via Eqs. (1), with reference to the overall CG. Thus, strictly, Eq. (26) should be replaced by
$$I_{ch} = \frac{\pi}{60}\rho D_0^5 B_i + m_{ch} L_c^2. \tag{5}$$
This amendment is implicit in White and Wolf's final expression for the dimensionless overall moment of inertia, i, in Eqs. (27);

- the 'conservative glide-stability criterion' presented in Eq. (29) requires the simplification $\alpha_0 = 0$, rather than $C_{T\alpha} = 0$ as stated. With $\alpha_0 = 0$, terms in $C_{T\alpha}$ do indeed disappear, because they all involve the factor $\sin\alpha_0$. However, setting $C_{T\alpha} = 0$ alone is insufficient because Eq. (29) also requires $\cos\alpha_0 = 1$;

- the derivation of Eq. (29) relies on the reduction of the characteristic equation to the cubic of Eq. (28). In fact, the characteristic equation remains quartic, but with one straightforwardly identifiable (and stable) root $\lambda = -C_{T0}/k$. The cubic form is the equation for the remaining three roots.

It is now possible to reconsider the results of the stability analysis, as presented in White and Wolf's Fig. 3. This plot has been recreated in accordance with the corrections set out above, and has been found accurate (within the limitations of the graphical representation). Hence it can be concluded that the errors in the original formulae are typographical, and the results as presented are reliable.

References

[1]White, F. M., and Wolf, D. F., "A Theory of Three-Dimensional Parachute Dynamic Stability," *Journal of Aircraft*, Vol. 5, No. 1, 1968, pp.86–92. doi: 10.2514/3.43912